EXERCISE – IV ADVANCED SUBJECTIVE QUESTIONS

1. Consider a triangle ABC with sides AB and AC having the equations $L_1 = 0$ and $L_2 = 0$. Let the centroid, orthocentre and circumcentre of the Δ ABC are G, H and S respectively. L = 0 denotes the equation of side BC.
(a) If $L_1 : 2x - y = 0$ and $L_2 : x + y = 3$ and G (2, 3) then find the slope of the line L = 0.

(b) If $L_1 : 2x + y = 0$ and $L_2 : x - y + 2 = 0$ and H(2, 3) then find the y-intercept of L = 0.

(c) If $L_1 : x+y - 1=0$ and $L_2 : 2x - y + 4=0$ and S(2, 1) then find the x-intercept of the line L = 0.

2. The equations of perpendiculars of the sides AB and AC of triangle ABC are $x-y-4=0$ and $2x-y-5=0$ respectively. If the vertex A is (–2, 3) and point of intersection of perpendiculars bisectors is $\left(\frac{3}{2},\frac{5}{2}\right)$, find the equation of medians to the sides AB and AC respectively.

3. The interior angle bisector of angle A for the triangle ABC whose coordinates of the vertices are A(–8, 5); B(–15, –19) and C(1, – 7) has the equation $ax+2y+c=0$. Find 'a' and 'c'.

4. Find the equation of the straight lines passing through (–2, –7) and having an intercept of length 3 between the straight lines $4x + 3y = 12$, $4x + 3y = 3$.

5. Two sides of a rhombus ABCD are parallel to the lines $y = x + 2$ and $y = 7x + 3$. If the diagonals of the rhombus intersect at the point (1, 2) and the vertex A is on the y-axis, find the possible coordinates of A.

6. A triangle is formed by the lines whose equations are AB : $x + y - 5 = 0$, BC : $x + 7y - 7 = 0$ and CA : $7x + y + 14 = 0$. Find the bisector of the interior angle at B and the exterior angle at C. Determine the nature of the interior angle at A and find the equation of the bisector.

7. A point P is such that its perpendicular distance from the line $y - 2x + 1 = 0$ is equal to its distance from the origin. Find the equation of the locus of the point P. Prove that the line $y = 2x$ meets the locus in two points Q and R, such that the origin is the mid point of QR.

8. Find the equations of the sides of a triangle having (4, –1) as a vertex, if the lines $x - 1 = 0$ and $x - y - 1 = 0$ are the equations of two internal bisectors of its angles.

9. P is the point (–1, 2), a variable line through P cuts the x and y axes at A and B respectively Q is the point on AB such that PA, PQ, PB are H.P. Show that the locus of Q is the line $y = 2x$.

10. The equations of the altitudes AD, BE, CF of a triangle ABC are $x + y = 0$, $x - 4y = 0$ and $2x - y = 0$ respectively. The coordinates of A are (t, –t). Find coordinates of B and C. Prove that if t varies the locus of the centroid of the triangle ABC is $x + 5y = 0$.

11. The distance of a point(x_1, y_1) from each of two straight lines which passes through the origin of co-ordinates is δ; find the combined equation of these straight lines.

12. Consider a Δ ABC whose sides AB, BC and CA are represented by the straight lines $2x + y = 0$, $x + py=q$ and $x - y = 3$ respectively. The point P is (2, 3).
(a) If P is the centroid, then find the value of (p + q).

(b) If P is the orthocentre, then find the value of (p + q).

(c) If P is the circumcentre, then find the value of (p + q).

13. Consider a line pair $2x^2+3xy-2y^2-10x+15y-28=0$ and another line L passing through origin with gradient 3. The line pair and line L form a triangle whose vertices are A, B and C.
(a) Find the sum of the cotangents of the interior angles of the triangle ABC.

(b) Find the are a of triangle ABC

(c) Find the radius of the circle touching all the 3 sides of the triangle.
14. Show that all the chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ which subtend a right angle at the origin are concurrent. Does this result also hold for the curve, $3x^2 + 3y^2 - 2x + 4y = 0$? If yes, what is the point of concurrency and if not, give reasons.

15. A straight line is drawn from the point (1, 0) to the curve $x^2 + y^2 + 6x - 10y + 1 = 0$, such that the intercept made on it by the curve subtends a right angle at the origin. Find the equations of the line.

16. The two line pairs $y^2 - 4y + 3 = 0$ and $x^2 + 4xy + 4y^2 - 5x - 10y + 4 = 0$ enclose a 4 sided convex polygon find

(i) area of the polygon

(ii) length of its diagonals.

17. Find the equation of the two straight lines which together with those given by the equation $6x^2 - xy - y^2 + x + 12y - 35 = 0$ will make a parallelogram whose diagonals intersect in the origin.